12 Areas

1 Area

The area of a figure measures the size of the region enclosed by the figure. This is expressed in one of the square units, square meters, square centimetres, square inches, and so on.

2 Quadrilaterals

2.1 Area of a square

If ‘a’ is the length of the side of a square, the area is $a \times a$ or a^2.

Example:

What is the area of a square if the side-length is 7cm?

The area is 7x7 =49 cm^2

2.2 Area of a rectangle

The area of a rectangle is the product of its width and length.

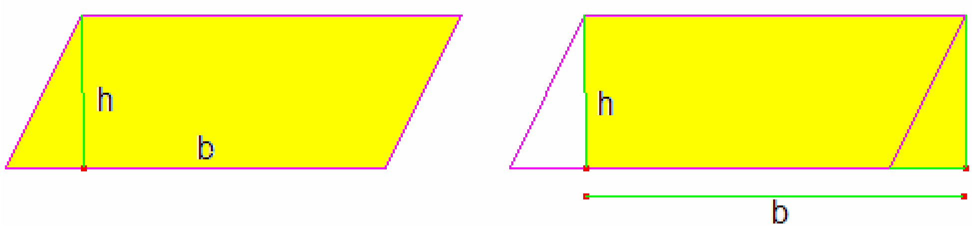
Example:

What is the area of a rectangle having a length of 12 m and a width of 3.2 m?

The area is the product of these two side-lengths, which is $12 \times 3.2 = 38.4$ m^2

2.3 Area of a parallelogram

The area of a parallelogram is $b \times h$, where b is the length of the base of the parallelogram, and h is the perpendicular height. To picture this, consider the parallelogram below:



We can cut a triangle from one side and paste it to the other side to form a rectangle with side-lengths b and h. the area of this rectangle is b × h.

Example:

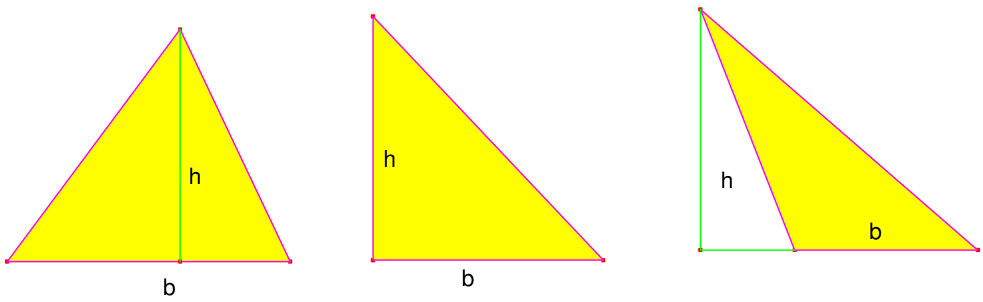
What is the area of a parallelogram with a base of 82 mm and a perpendicular height of 73 mm?

The area $A = 82 \times 73 = 5986\ \text{mm}^2$

3 Area of a triangle

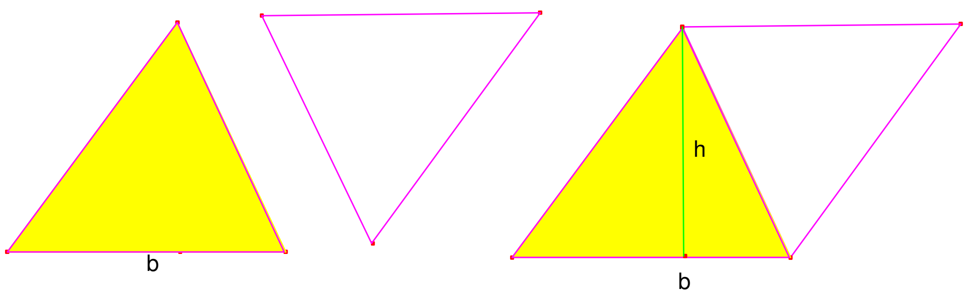
Consider a triangle with base length *b* and height *h*.

The area of the triangle is $\frac{1}{2} b \times h$.



The perpendicular height can be inside the triangle, one of its sides or outside of the triangle as can be seen in the picture.

To demonstrate this formula, we could take a second triangle identical to the first, then rotate it and "paste" it to the first triangle as pictured below:

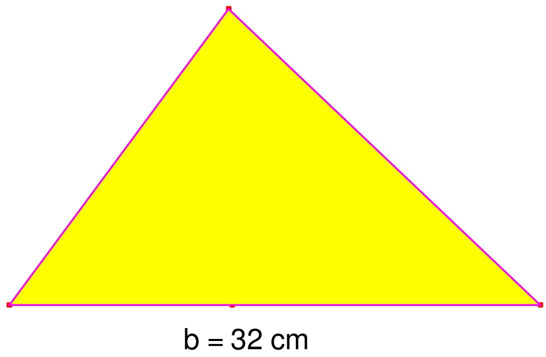


The figure formed is a parallelogram with base length *b* and height *h*, and has area $b \times h$.

This area is twice that the area of the triangle, so the area of the triangle is $\frac{1}{2}b \times h$

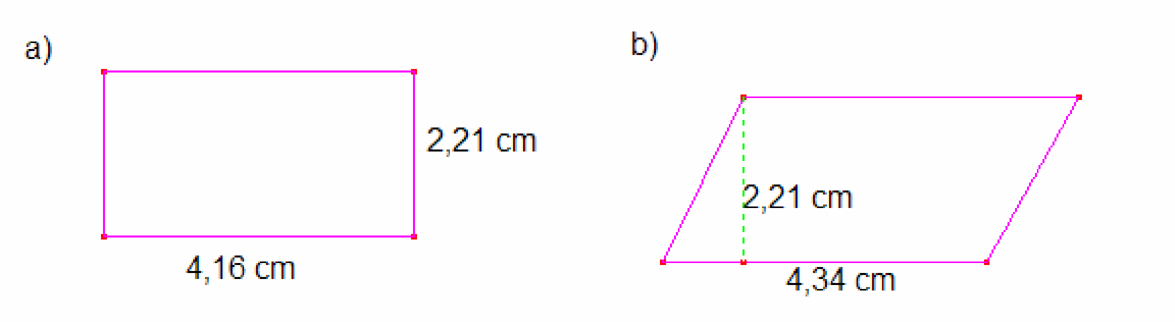
Example:

What is the area of the triangle of the picture on the right?

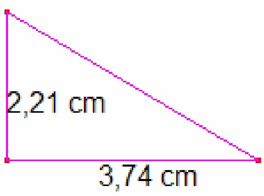


The area of a triangle is half the product of its base and height, which is $A = \frac{1}{2}15 \times 32 = 240 \text{ cm}^2$

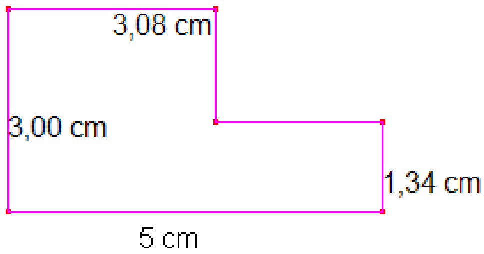
Exercise 1 Calculate the area of these shapes



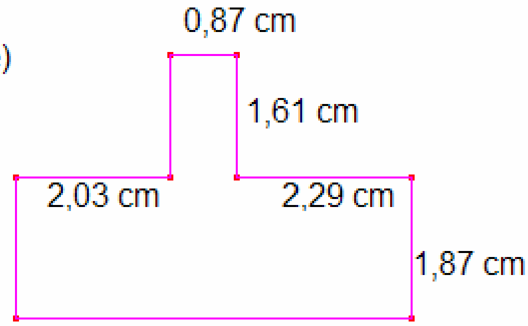
c)


2,21 cm
3,74 cm

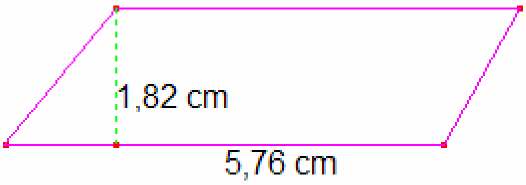
d)


3,08 cm
3,00 cm
1,34 cm
5 cm

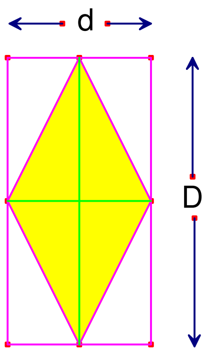
e)


0,87 cm
1,61 cm
2,03 cm
2,29 cm
1,87 cm

f)


1,82 cm
5,76 cm

4 Area of a rhombus



The area of a rhombus can be calculated as the area of a parallelogram as we have seen in point 2.3, but it can also be calculated if we know the length of the diagonals.

The area of the rectangle is $D \times d$, there are four equal triangles inside and outside the rhombus, so the area of the rhombus is $A = \frac{D \times d}{2}$.

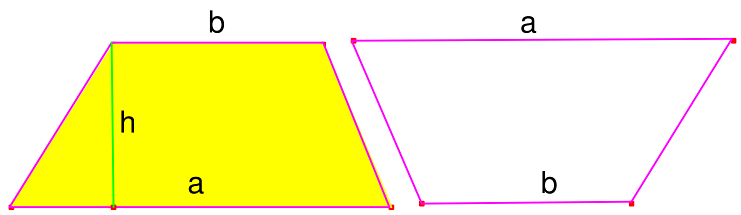
The perimeter of a rhombus is $4l$ if l is the length of each side.

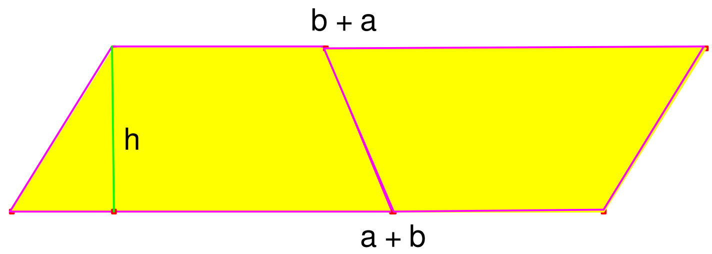
Exercise 2 Calculate the area and the perimeter of a rhombus, whose diagonals are of 6cm and 8 cm.

5 Area of a trapezium

If a and b are the lengths of the two parallel bases of a trapezium, and h is its height, the area is $A = \frac{a+b}{2} \cdot h$.

To demonstrate this, consider two identical trapezoids, and "turn" one around and "paste" it to the other along one side as it is drawn below:





The figure formed is a parallelogram having an area of $A = (a+b)\cdot h$, which is twice the area of one trapezium.

E*xample:*

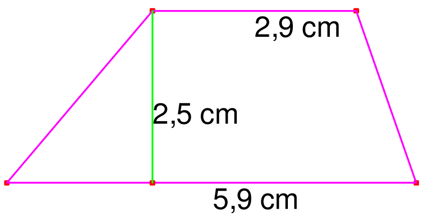
What is the area of a trapezium having bases 13 and 9 and a height of 6?

Using the formula for the area of a trapezoid, we see that the area is

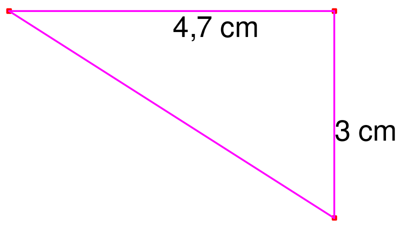
$\frac{13+9}{2}\times 6 = 66$ units of area

Exercise 3 Calculate the area of these shapes

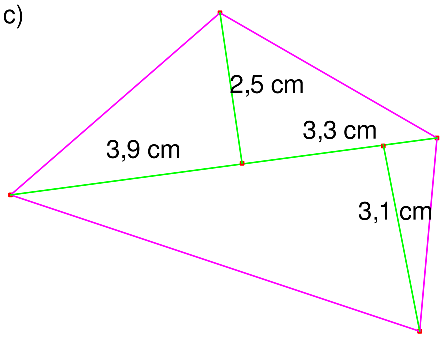
a)



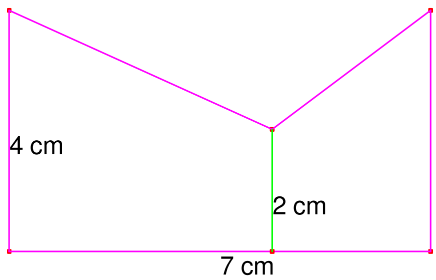
b)



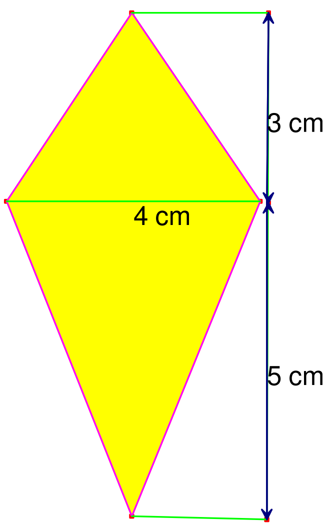
c)


2,5 cm
3,3 cm
3,9 cm
3,1 cm

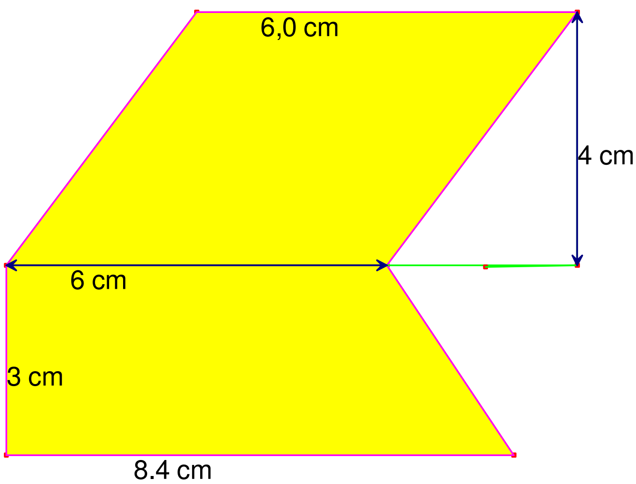
d)


4 cm
2 cm
7 cm

e)


3 cm
4 cm
5 cm

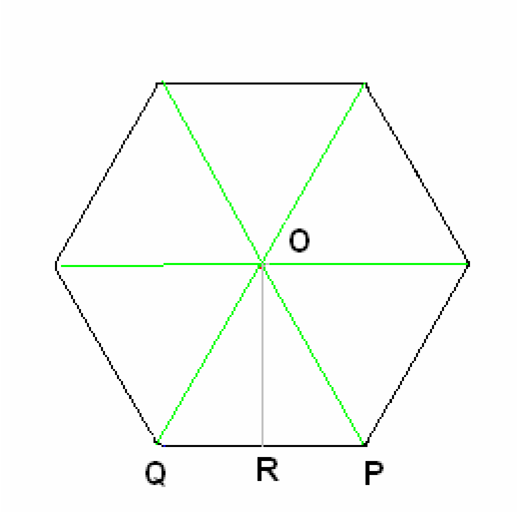
f)


6,0 cm
4 cm
6 cm
3 cm
8.4 cm

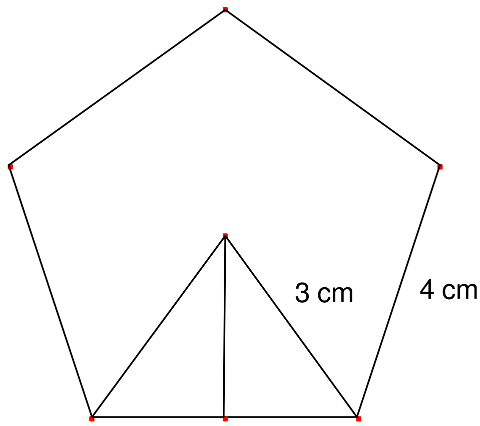
6 Measures in regular polygons and circles

6.1 Area and perimeter of regular polygons

All the regular polygons can be divided into n equal isosceles triangles and in any of these triangles the base is the side of the polygon (like QP) and the height is the apothem OR, the area of each triangle is $A = \frac{\text{poligon side} \times \text{apotheme}}{2}$ so the area of a regular polygon with n sides of length l and apothem a is. $A = \frac{l \times a}{2} n = \frac{p \times a}{2}$ Where p is the perimeter



Exercise 4 Calculate the apothem of this pentagon and the its area



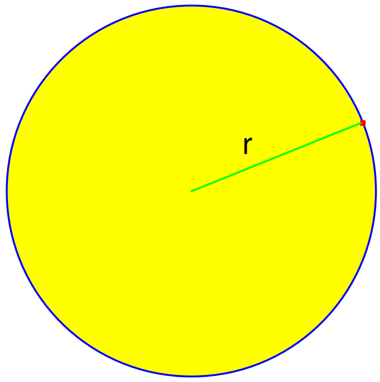
Measures in a circle

6.2 Area of a circle

The area of a circle is $\pi \times r^2$, where *r* is the length of its radius. π is a number that is approximately 3.14159.

Example:

What is the area of a circle having a radius of 53 cm, to the nearest tenth of a square cm? Using an approximation of 3.1415927 for π, and the fact that the area of a circle is $\pi \times r^2$, the area of this circle is $3.1415927 \times 53^2 = 8824.73$ square cm, which is 8824.7 cm^2 rounded to the nearest tenth.



6.3 Length of a circumference

The length of a circle's circumference is $l = 2\pi \cdot r$

Example:

The length of the circumference of the previous circle using for π 3.14 is $l = 3.14 \times 53 = 166.4\text{cm}$

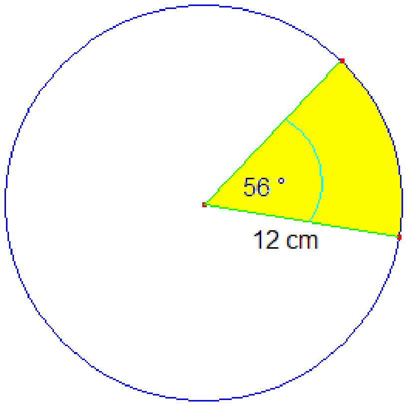
6.4 Length of an arc of the circumference

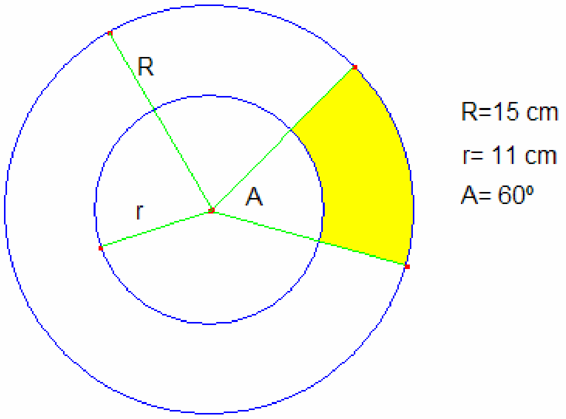
The length of an arc of the circumference of n degrees is $l = \frac{2\pi r}{360} n$ because there is a direct proportionality between the length of the arc and the corresponding angle.

6.5 Area of a sector

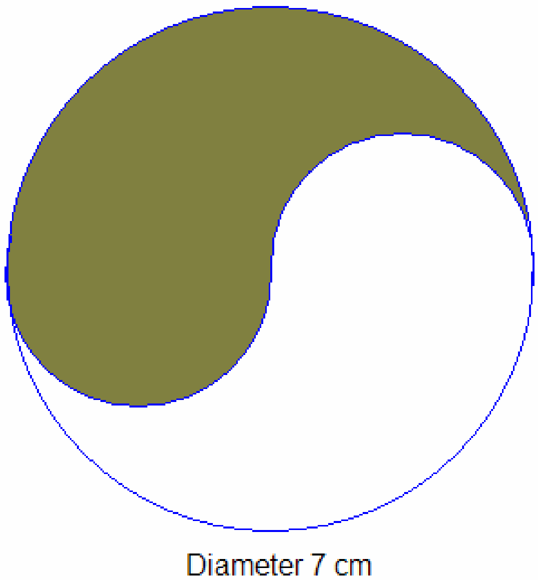
There is also a direct proportionality between the area of a sector and the corresponding angle, so $A = \frac{\pi \cdot r^2}{360} n$.

Exercise 5 Calculate the length or the arcs and the area of the shaded part in the pictures.

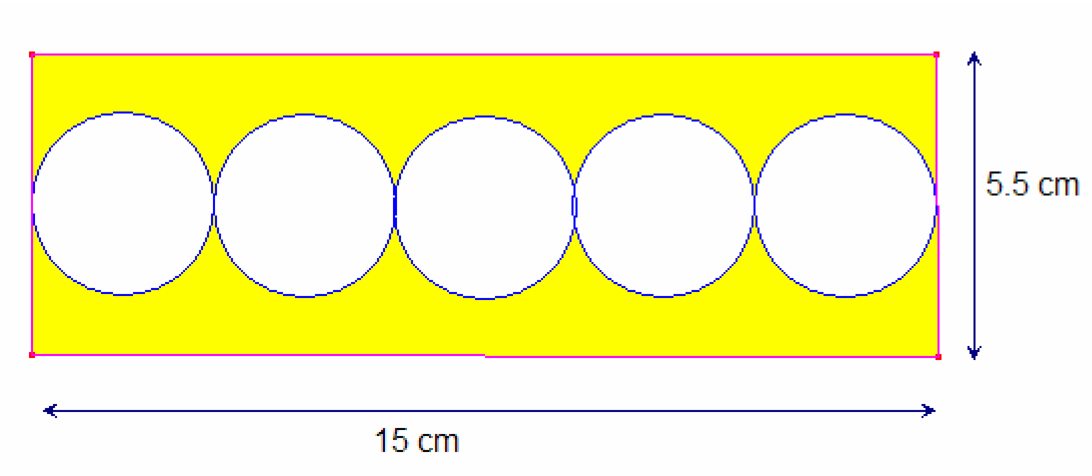


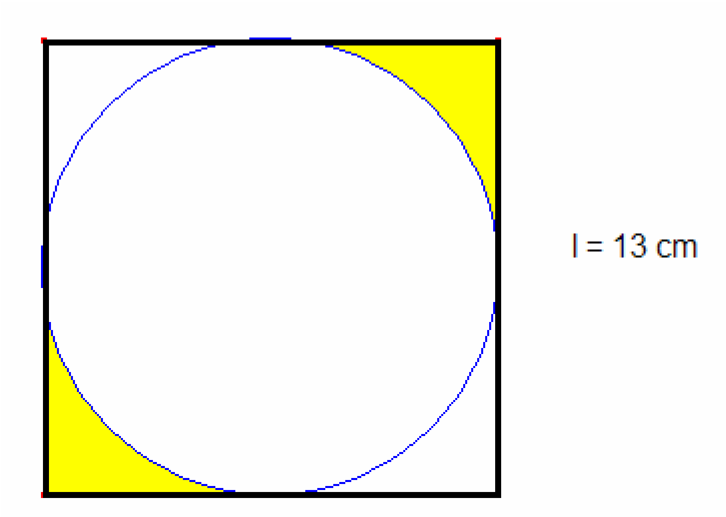


Exercise 6 Calculate the area of the shaded part in this logo.



Exercise 7 Calculate the area of the shaded part in these pictures.


5.5 cm
15 cm


l = 13 cm

Solutions

Exercise 1 a) $9.19\,cm^2$, b) $9.59\,cm^2$, c) $4.13\,cm^2$, d) $12.43\,cm^2$, e) $11.11\,cm^2$, f) $10.48\,cm^2$.

Exercise 2 $24\,cm^2$

Exercise 3 a) $11cm^2$, b) $7.05\,cm^2$, c) $20.16\,cm^2$, d) $21cm^2$, e) $16\,cm^2$, f) $45.6\,cm^2$.

Exercise 4 Apothem $2.24\,cm$, area $22.36\,cm^2$

Exercise 5 Left picture: Arc L = 11.73 cm, Area = 70.37 cm^2. Right picture Arcs L = 15.71 cm, l= 11.52 cm, Area $A = 117.8 - 63.36 = 54.45\,cm^2$.

Exercise 6 $A = 19.24\,cm^2$

Exercise 7 Picture 1: $A_{circles} = 35.34\,cm^2$, shaded area $A = 47.16\,cm^2$**.** Picture 2: Square area $A_{sq} = 169\,cm^2$, circle area $A_c = 132.73\,cm^2$, shaded area $A = 18.13\,cm^2$

12 Areas

1 Area

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2 Quadrilaterals

2.1 Area of a square

If ‘a’ is the length of the side of a square, the area is $a \times a$ or a^2.

Example:

What is the area of a square if the side-length is 7cm?

The area is 7x7 =49 cm^2

2.2 Area of a rectangle

The area of a rectangle is the product of its width and length.

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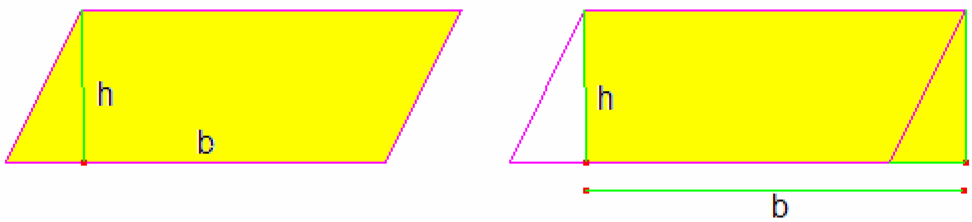
The area of a parallelogram is $b \times h$, where b is the length of the base of the parallelogram, and h is the perpendicular height. To picture this, consider the parallelogram below:



We can cut a triangle from one side and paste it to the other side to form a rectangle with side-lengths b and h. the area of this rectangle is b × h.

Example:

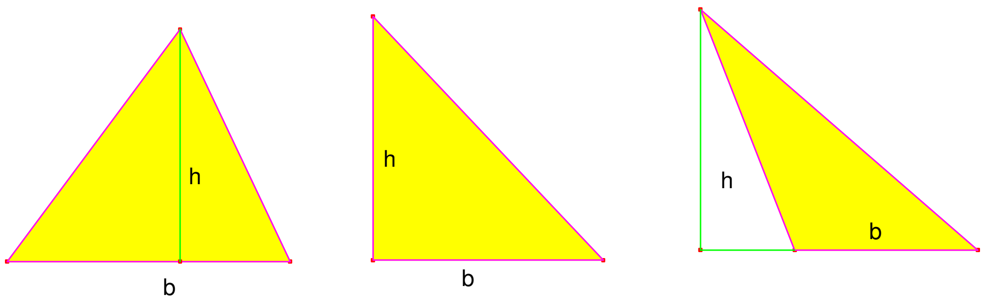
What is the area of a parallelogram with a base of 82 mm and a perpendicular height of 73 mm?

The area $A = 82 \times 73 = 5986 \text{ mm}^2$

3 Area of a triangle

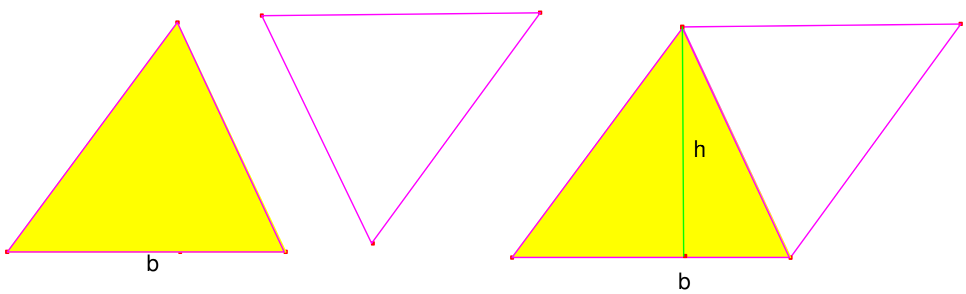
Consider a triangle with base length *b* and height *h*.

The area of the triangle is $\frac{1}{2} b \times h$.



The perpendicular height can be inside the triangle, one of its sides or outside of the triangle as can be seen in the picture.

To demonstrate this formula, we could take a second triangle identical to the first, then rotate it and "paste" it to the first triangle as pictured below:



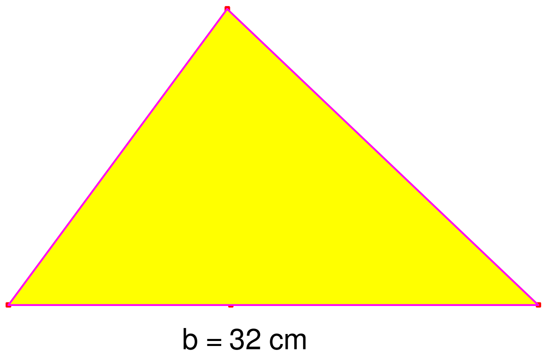
The figure formed is a parallelogram with base length *b* and height *h*, and has area $b \times h$.

This area is twice that the area of the triangle, so the area of the triangle is

$\frac{1}{2}b\times h$

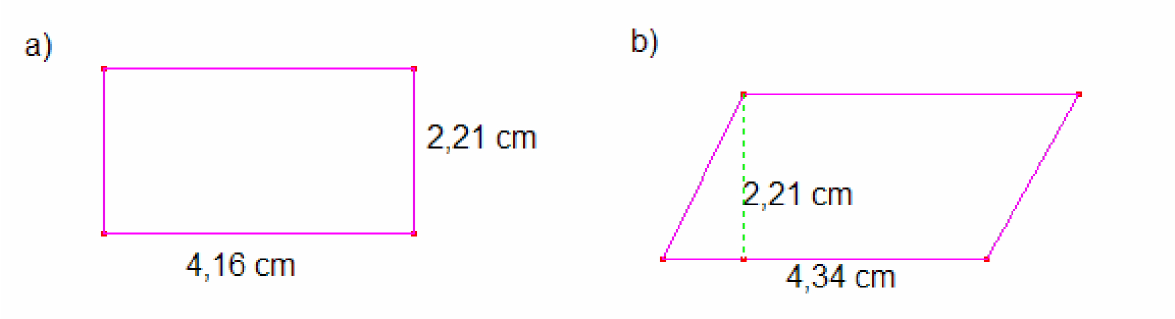
Example:

What is the area of the triangle of the picture on the right?

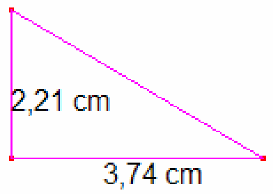


The area of a triangle is half the product of its base and height, which is $A=\frac{1}{2}15\times32=240\text{ cm}^2$

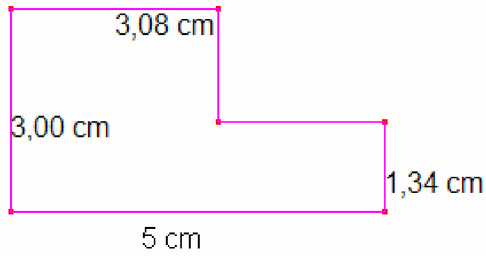
Exercise 1 Calculate the area of these shapes



c)


2,21 cm
3,74 cm

d)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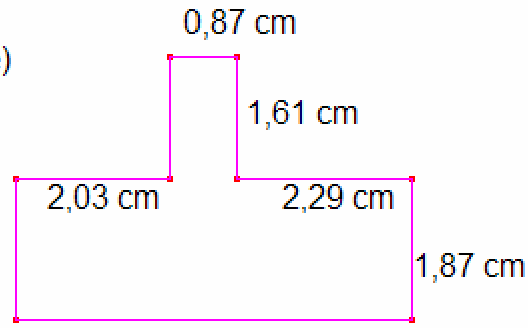
3,00 cm
1,34 cm
5 cm

e)


0,87 cm
1,61 cm
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2,29 cm
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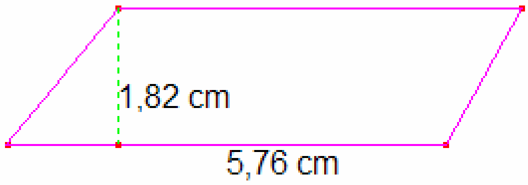
1,82 cm
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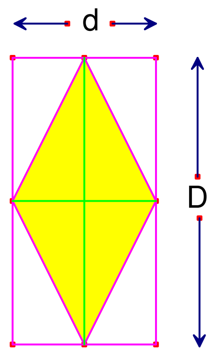
4 Area of a rhombus



The area of a rhombus can be calculated as the area of a parallelogram as we have seen in point 2.3, but it can also be calculated if we know the length of the diagonals.

The area of the rectangle is $D \times d$, there are four equal triangles inside and outside the rhombus, so the area of the rhombus is $A = \frac{D \times d}{2}$.

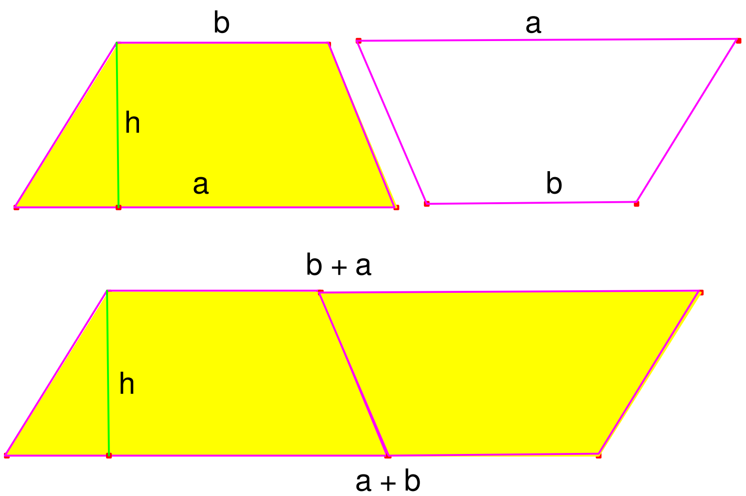
The perimeter of a rhombus is $4l$ if l is the length of each side.

Exercise 2 Calculate the area and the perimeter of a rhombus, whose diagonals are of 6cm and 8 cm.

5 Area of a trapezium

If a and b are the lengths of the two parallel bases of a trapezium, and h is its height, the area is $A = \frac{a+b}{2} \cdot h$.

To demonstrate this, consider two identical trapezoids, and "turn" one around and "paste" it to the other along one side as it is drawn below:



The figure formed is a parallelogram having an area of $A = (a+b)\cdot h$, which is twice the area of one trapezium.

E*xample:*

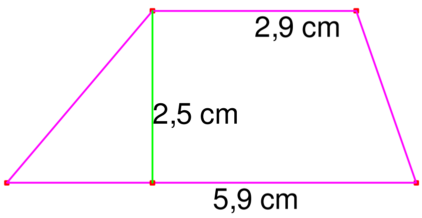
What is the area of a trapezium having bases 13 and 9 and a height of 6?

Using the formula for the area of a trapezoid, we see that the area is

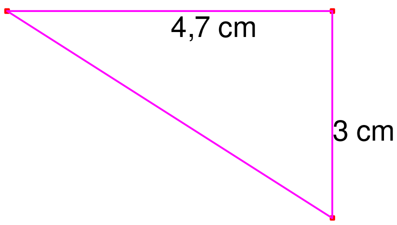
$\frac{13+9}{2} \times 6 = 66$ units of area

Exercise 3 Calculate the area of these shapes

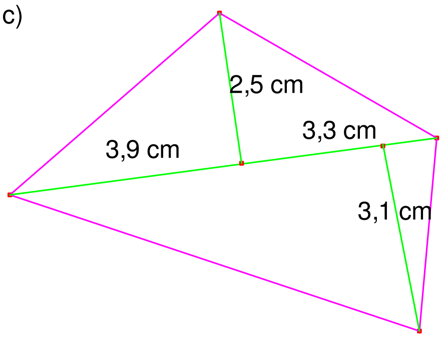
a)



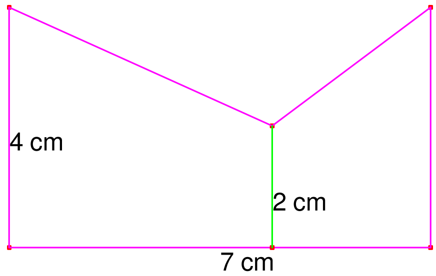
b)



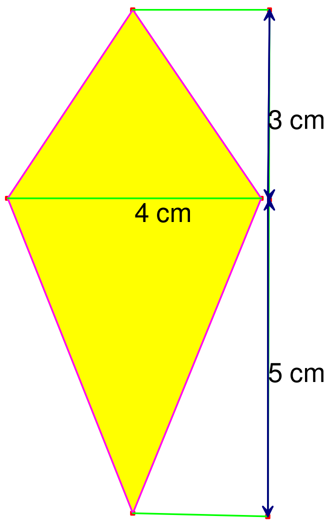
c)


2,5 cm
3,3 cm
3,9 cm
3,1 cm

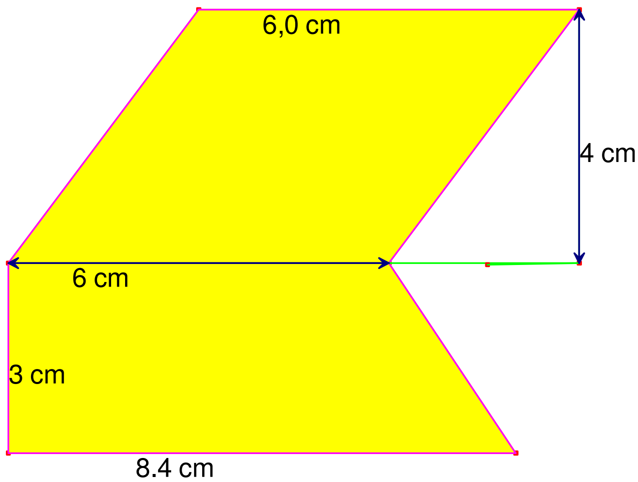
d)


4 cm
2 cm
7 cm

e)


3 cm
4 cm
5 cm

f)

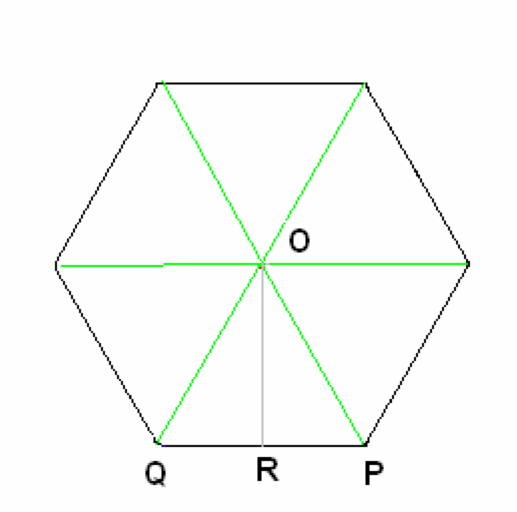

6,0 cm
4 cm
6 cm
3 cm
8.4 cm

6 Measures in regular polygons and circles

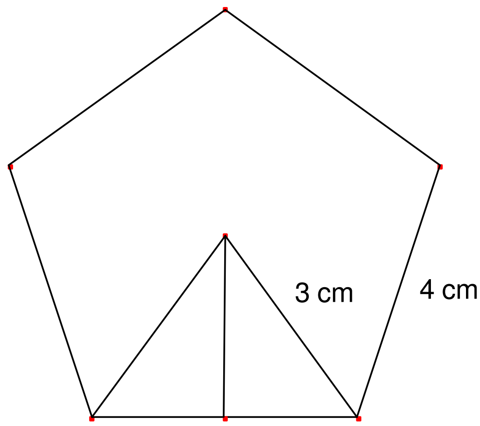
6.1 Area and perimeter of regular polygons

All the regular polygons can be divided into n equal isosceles triangles and in any of these triangles the base is the side of the polygon (like QP) and the height is the apothem OR, the area of each triangle is $A = \frac{\text{poligon side} \times \text{apotheme}}{2}$ so the area of a regular polygon with n sides of length l and apothem a is.

$A = \frac{l \times a}{2} n = \frac{p \times a}{2}$ Where p is the perimeter



Exercise 4 Calculate the apothem of this pentagon and the its area



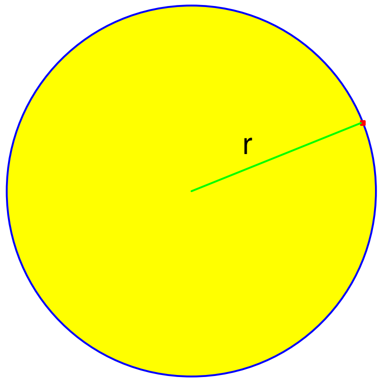
Measures in a circle

6.2 Area of a circle

The area of a circle is $\pi \times r^2$, where *r* is the length of its radius. π is a number that is approximately 3.14159.

Example:

What is the area of a circle having a radius of 53 cm, to the nearest tenth of a square cm? Using an approximation of 3.1415927 for π, and the fact that the area of a circle is $\pi \times r^2$, the area of this circle is $3.1415927 \times 53^2 = 8824.73$ square cm, which is 8824.7 cm^2 rounded to the nearest tenth.



6.3 Length of a circumference

The length of a circle’s circumference is $l = 2\pi \cdot r$

Example:

The length of the circumference of the previous circle using for π 3.14 is $l = 3.14 \times 53 = 166.4cm$

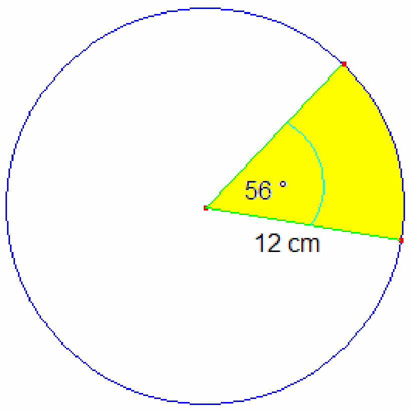
6.4 Length of an arc of the circumference

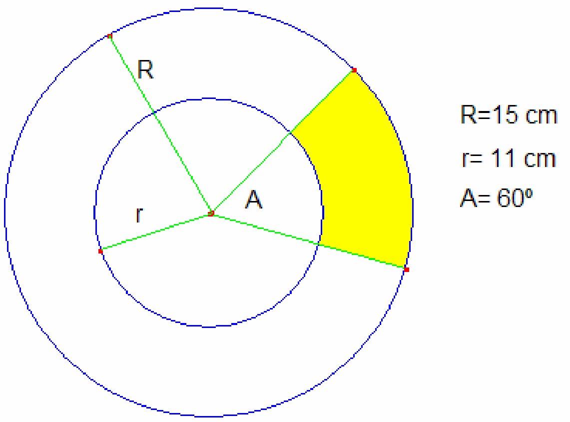
The length of an arc of the circumference of n degrees is $l = \frac{2\pi r}{360} n$ because there is a direct proportionality between the length of the arc and the corresponding angle.

6.5 Area of a sector

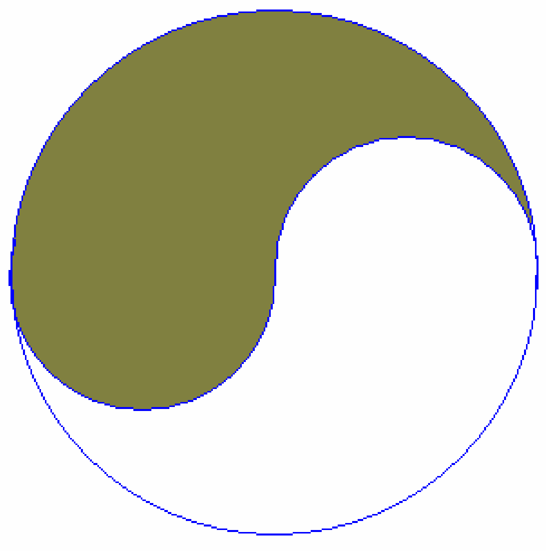
There is also a direct proportionality between the area of a sector and the corresponding angle, so $A = \frac{\pi \cdot r^2}{360} n$.

Exercise 5 Calculate the length or the arcs and the area of the shaded part in the pictures.

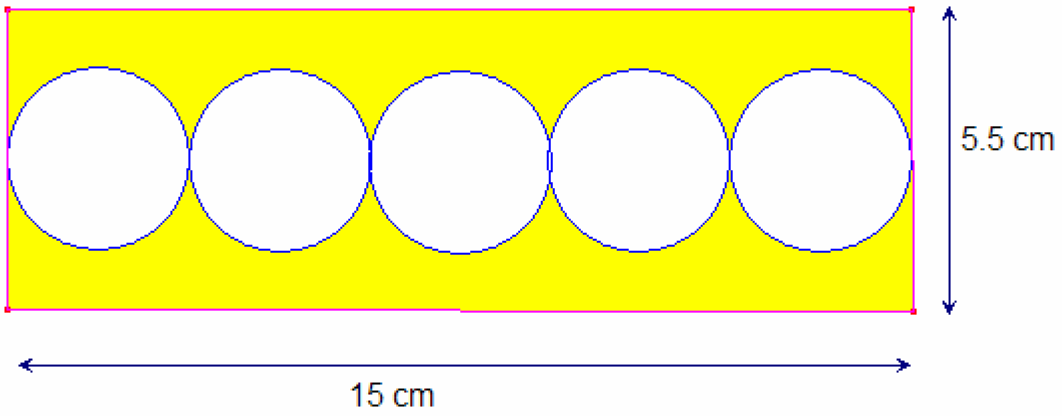


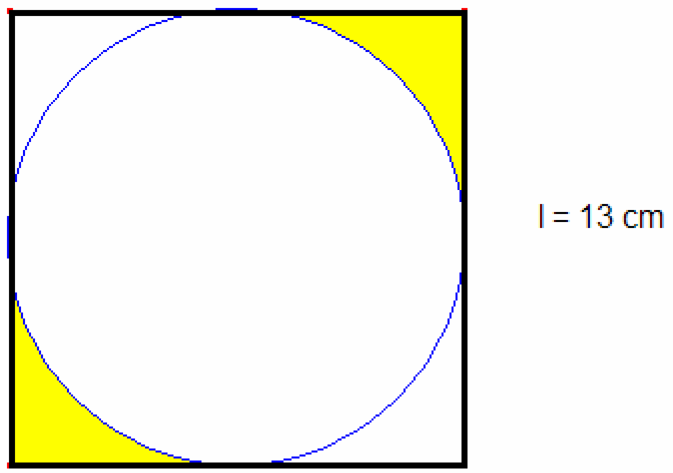


Exercise 6 Calculate the area of the shaded part in this logo.



Exercise 7 Calculate the area of the shaded part in these pictures.


5.5 cm
15 cm


l = 13 cm

Solutions

Exercise 1 a) $9.19\,cm^2$, b) $9.59\,cm^2$, c) $4.13\,cm^2$, d) $12.43\,cm^2$, e) $11.11\,cm^2$, f) $10.48\,cm^2$.

Exercise 2 $24\,cm^2$

Exercise 3 a) $11cm^2$, b) $7.05\,cm^2$, c) $20.16\,cm^2$, d) $21cm^2$, e) $16\,cm^2$, f) $45.6\,cm^2$.

Exercise 4 Apothem $2.24\,cm$, area $22.36\,cm^2$

Exercise 5 Left picture: Arc L = 11.73 cm, Area = 70.37 cm^2. Right picture Arcs L = 15.71 cm, l= 11.52 cm, Area $A = 117.8 - 63.36 = 54.45\,cm^2$.

Exercise 6 $A = 19.24\,cm^2$

Exercise 7 Picture 1: $A_{circles} = 35.34\,cm^2$, shaded area $A = 47.16\,cm^2$.
Picture 2: Square area $A_{sq} = 169\,cm^2$, circle area $A_c = 132.73\,cm^2$, shaded area $A = 18.13\,cm^2$